Quantifying flow: changing the mathematics underlying neural synchronisation

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The possible correlation between a state of flow in a team and neural synchronisation between the individuals in the team is a fascinating research topic. However, it is essential that the analytical technique to quantify neural synchronisation does not give false positives. This paper explores the mathematics behind the Phase Locking Index and gives an argumentation why this technique in its current form could lead to false positives. A change in the mathematical approach is proposed, such that a Neural Synchronisation Vector is obtained that can avoid this type of error.

Keywords: neural synchronisation, EEG, hyperscanning, phase locking index, coupled oscillators

Introduction

Cooperation is an essential part of the human existence. We are constantly tuning in to others, anticipating their actions and adapting our own. This can be as simple as participating in traffic or waiting in a queue and as complex as the coordination of international projects. If we want our actions to be optimally aligned, we need everyone in the team to be in tune with each other. While a high quality of teamwork might be most directly visible in the performance of sports teams, dancers and musician it might be even more crucial in places like the boardrooms of large organisations or the controlrooms of crucial technical operations like powerplants or air traffic control. The need to improve cooperation, communication and flow between teams of people has lead to a multitude of books, workshops, trainers, coaches and therapists that promise to improve these things. What actually defines good teamwork is often considered to be something subjective. People have the feeling they are "in sync" with each other, or can reach a certain level of "flow", but these states are often considered to have some kind of magical aura that can not be measured. And even though there are attempts to objectify this state through psychological models or even mathematical looking formulas that describe how we should combine certain psychological qualities in order to maximize a state of creative flow when working together (Nakamura & Csikszentmihalyi, 2014; Csikszentmihalyi, 2015; Gratton, 2007), none of these are able to objectively quantify teamwork. While this does not necessarily have to be a problem and might even the preferred option for some people or contexts, it can be useful to complement the subjective assessment of the state of flow with an objective quantification. First, objective feedback about the quality of teamwork could reinforce good practice in the same way neurofeedback for individuals can help them to improve traits like concentration (Arns, de Ridder, & Strehl, 2009; Arns, Drinkenburg, & Leon Kenemans, 2012; Fingelkurts, Fingelkurts, & Kallio-Tamminen, 2015; Hammond, 2007; Surmeli & Ertem, 2010). Secondly, it can be difficult to "see the water you are swimming in as a fish" and we do not always have easy access to an outsider that can help us reflect on the quality of our interaction. Thirdly, we can enjoy learning about the extend to which our brains act like coupled oscillators for the inherent joy of learning itself. We can find the phenomena of coupled oscillators throughout physics and biology, for example in the movement of pendulums or planets and inside our body as the pacemaker cells of the heart or our spinal cord that controls breathing, running and chewing. But it is also common to find coupled oscillators between organisms, for example in the chirping of crickets or the synchronous flashing of fireflies as illustrated in figure 1 (Strogatz & Stewart, 1993). The first section of this paper will explore hyperscanning studies that aim to correlate neural synchronisation with teamwork. Possible pitfalls in the research designs are discussed, followed by a selection of mathematical techniques used to quantify neural synchronisation. The second section argues under which conditions the quantification of neural synchronisation with the Phase Locking Index could lead to false positives and suggests the alternative mathematical approach of the Neural Synchronisation Vector in order to avoid this type of error.

Hyperscanning studies

Research designs

The analysis of the brainsignals of more than one person at a time is called 'hyperscanning' and can be done with methods like fMRI or EEG (Sänger, Lindenberger, &

Figure 1. Synchronous flashing fireflies. Image Credit: Radim Schreiber

Müller, 2011). This paper will mainly focus on the analysis of EEG signals, because of the higher time-resolution as compared to fMRI and the easier access to EEG devices which makes the research easier to execute. In the last decade there have been several studies that used the hyperscanning paradigm to research the hypothesis that phase synchronisation between brains is a neural substrate for teamwork. Neural synchronisation during teamwork is dominantly measured by the Phase Locking Index (PLI), an analytical technique that which will be discussed in depth in the next section. A significant increase of the PLI and similar measures have been found under a wide variety of research designs including the formation of spontaneous leader-follower pairs during group discussion (Shi et al., 2015), joint attention in a visual search task (Szymanski et al., 2017), cooperation on a puzzle task (Cha & Lee, 2018), spontaneous synchronisation of hand movements (Dumas, Nadel, Soussignan, Martinerie, & Garnero, 2010; Delaherche, Dumas, Nadel, & Chetouani, 2015), the degree of cooperation between pilots in different phases of a flight (Toppi et al., 2016), guitarists engaged in musical improvisation (Müller, Sänger, & Lindenberger, 2013) and guitarists playing melodies together (Lindenberger, Li, Gruber, & Müller, 2009; Sänger, Müller, & Lindenberger, 2012). While this is a wide spectrum of settings in which significant results are measured, some of these research designs can be critized for the lack of proper control conditions where the social aspect is missing while the aspects of perceptual input and motor output are kept constant (Szymanski et al., 2017). After all, it might be a very real possiblity that synchronised hand movements or simultaneously playing the same music have a neural substrate that is interpreted as neural synchronisation, while what is actually measured is that people perform the same activity. Possibly the most convincing design to counter this critique is the research of Szymanski et al. (2017) that used joint attention in a visual search task as the only independent variable while keeping all other aspects like the visual input and the physical activity of the participants constant. Another question that surfaces when looking at the different research designs is how to define teamwork and cooperation. Improvising music together is a very different activity as compared to solving a puzzle together, which in turn is very different from making simultaneous hand movements. While this is not necessarily a problem, we might be measuring very different phenomena and turn out to be generalising over activities that should be considered as different findings.

Mathematical approaches to quantify neural synchronisation

The EEG signal is an oscillating, electric signal that has been measured by electrodes placed on the scalp since the 1930s (Bruch, 1959). To analyse this signal, a lot of different analytical techniques have been used over the decades for different purposes. An important aspect to consider when researching neural synchronisation is the mathematical approach that is used to quantify what is measured. There are multiple approaches to analyse statistical properties of coupled oscillators that make sense under different circumstances and are often originally developed for completely different situations than the EEG signal. The next section will discuss three techniques used to quantify the EEG signal: (i) power spectral analysis (ii) bispectrum analysis (iii) phase locking index.

Power spectral analysis. Even though it is not possible to quantify neural synchronisation with power spectral analysis, this approach will be discussed here shortly. This analytical technique is interesting because the results of some of the studies using this technique have implications for how we approach neural synchronisation mathematically, as will be make clear in the last part of the paper. Also, the limitations of this analysis give us a perspective on what we actually want to measure when considering neural synchronisation. The power spectral analysis quantifies the amplitude of the signal as a function of the frequency. The EEG signal is an electric signal that oscilates, seemingly chaotic. These chaotic oscillations measured at the skin are in fact the summation of a large amount of individual neurons firing inside the brain (Bruch, 1959; Ward, 2003; David & Friston, 2003). A first step towards analysing the signal is to decompose it into a summation of sinusoids, known as a Fourier series (Sigl & Chamoun, 1994). While we started with a single but seemingly chaotic signal we now have multiple sinusoids of different frequencies. This representation makes it much easier to study the signal. If we analyse the amplitudes of the sinusoids in the Fourier series as a function of the frequency, we have created the power spectrum. It is now possible to draw conclusions about which frequency band is the most dominant in specific parts of the brain and how the distribution of the frequencies changes over time or correlates with states of mind and activities. This approach turned out to be able to show significant correlations between a wide variety of things, for example the phases of sleep (Aeschbach & Borbély, 1993; Cajochen, Foy, & Dijk, 1999), depth of meditation (Fingelkurts et al., 2015; Fingelkurts, Fingelkurts, & Kallio-Tamminen, 2016; Travis et al., 2009), phenotyping of ADHD (Arns, Gunkelman, Breteler, & Spronk, 2008; Arns et al., 2009, 2012; Arns, Heinrich, & Strehl, 2014), predicting the effectivity of different types of medication for conditions like ADHD or depression (John, Prichep, & Almas, 1992; Johnstone, Gunkelman, & Lunt, 2005), stages of problemsolving and creativity (Sandkühler & Bhattacharya, 2008; Dietrich & Kanso, 2010), creative ideation (Fink & Benedek, 2014), improvisation dancing (Fink, Graif, & Neubauer, 2009), drawing or claying (Kruk, Aravich, Deaver, & Debeus, 2014) and the speed of acquisition of a second language (Prat, Yamasaki, Kluender, & Stocco, 2016). These studies have established the existence of a significant correlation between specific states of mind and specific frequency bands in the EEG signal. The whole domain of neurofeedback exploits this correlation by reinforcing people when they manage to get their brains withing a certain frequency band which actually leads to interesting results like a reduction in the symptoms of ADHD (Hammond, 2007; Arns et al., 2014). From research like this we can draw two important conclusions: (i) specific states of mind and activities imply that the brain oscillates more within specific frequency bands (ii) training the brain to oscillate within certain frequency bands through neurofeedback induces certain states of mind. While the power spectral analysis has become one of the dominant techniques applied in the domain of neurofeedback (Hammond, 2007) it also has some important limitations. One general problem is that it assumes that the EEG signal arises from a linear process and ignores possible interactions between components of the signal, which could be problematic because almost all biological systems show nonlinear behavior (Sigl & Chamoun, 1994). In addition to this, the power spectral analysis ignores all phase information in the signal. Because only the amplitude of the sinusoids is taken into account, the power spectral analysis can not discern between signals that have a different phase or are synchronised. Because complex functions of the brain require different parts from the brain to cooperate, we can gain important additional information about these processes when we include the phase synchronisation in the analysis of the signal (Ward, 2003). There have been different attempts to incorporate the phase information of a signal into the analysis of the EEG signal.

Bispectral analysis. One apporach that takes the phase information of the EEG into account is the bispectral analysis, a type of analysis that was originally introduced by geophysicists to study phenomena like ocean waves, seismic activity and sunspots (Sigl & Chamoun, 1994). The bispectral analysis was intended to analyse one signal that is the result of different frequencies that interact in a nonlinear fashion (Hagihira, Takashina, Mori, Mashimo, & Yoshiya, 2001; Sigl & Chamoun, 1994). In order to do a bispectral analysis the signal is first decomposed into a Fourier series that is defined as:

$$X(f) = 2/M \sum_{k=0}^{M-1} x(k)e^{-ik2\pi f} \tag{1}$$

where the samples of the signal x are denoted as $x(k)$ for $k = \{0, \ldots, M-1\}$ and where M are the total amount of samples in the segment of data. i is the complex number $\sqrt{-1}$, f is a particular frequency component that can be described as $f \in \mathcal{F}$ where $\mathcal{F} = \{0, \ldots, f_s/2Hz\}$, and f_s is the sampling rate in samples per second (Sigl & Chamoun, 1994). To compute the bispectrum, the signal is divided into epochs and the Fourier transform of every epoch is computed. Then every frequency $f_i \in \mathcal{F}$ is combined with another frequency

$f_j \in \mathcal{F}$ to obtain the bispectrum of a couple f_i, f_j):

$$B(f_i, f_j) = \left| \sum_{n=1}^{L} X_n(f_i) X_n(f_j) X_n * (f_1 + f_j) \right| \tag{2}$$

where $X_n * (f_i + f_j)$ is the complex conjugate of $X_n(f_i + f_j)$, the subscript n refers to the epoch number and $X_n(f)o$ is the Fourier transform of the n-th epoch with a total of L epochs. The result of this analysis is a mapping of the combination of two frequencies to a real number: $\mathcal{F} \times \mathcal{F} \rightarrow \mathbb{R}$. One of the successful applications of the bispectrum with the EEG has been to correlate this measure to the depth of anesthesia (Avidan et al., 2008; Sebel et al., 1997; Sigl & Chamoun, 1994; Liu, Singh, & White, 1996). Recently, the bispectrum has been the proposed method in a hyperscanning study to measure the quality of teamwork (Cha & Lee, 2018). I have several points of critique on using the bispectrum analysis to measure neural synchronisation. (i) The bispectrum analysis is developed to be used on the Fourier transform of just one signal. To use it to compare two signals, you would have to modify the formula slightly. It is not clearly described how (Cha & Lee, 2018) precisely did this. Two signals would produce two sets of frequencies $\mathcal{F}_1$ and $\mathcal{F}_2$ that would produce a set of possible combinations $\mathcal{F}_1 \times \mathcal{F}_1 \times \mathcal{F}_2 \times \mathcal{F}_2$, which would require a fivedimensional space if we would map these combinations to $\mathbb{R}$. Somehow Cha and Lee (2018) managed to reduce the combinations of these two signals to a single, real-valued number as a function of the passing time. To judge the significance of their approach, it should be clear how they precisely did this. (ii) The bispectral analysis takes the amplitude of the signal into account, which might give unexpected results when we simply try to compare the amount to which the phases of independent signals are synchronised. (iii) The bispectral analysis is developed to show nonlinear couplings between the components of one signal. It is not a simple task to exactly understand what is going on when we apply this technique to two signals, especially because the computation involves a triple product, a complex conjugate and a fivedimensional space to visualise the combination of two signals. Even if it would be reproducable that the bispectral analysis has a significant correlation with the level of teamwork, it is hard to understand what we are measuring exactly and how we should interpret such results.

Phase Locking Index. A much simpler and more direct way of computing neural synchronisation is the Phase Locking Index (PLI) (Chavez, Le Van Quyen, Navarro, Baulac, & Martinerie, 2003; Boon et al., 2009). Confusingly, this technique is referred to by different authors as respectively the "Mean Phase Coherence", "Phase Locking Value", "intensity of the first Fourier mode of the Phase Distribution" and "Phase Locking Index" (Boon et al., 2009). Because there are slight differences between the authors in denoting the mathematical formula, it takes quite some effort to identify whether the same formula has been used by two authors under a different name, or if they actually use another technique. There is at least one case where an author compared two studies and supposed two analytical techniques (namely, the Phase Locking Index versus the Phase Locking Value) were different techniques (Sänger et al., 2011), while they actually are the same mathematical technique (Lindenberger et al., 2009; Dumas et al., 2010; Boon et al., 2009). I will adopt the nomenclature "Phase Locking Index" as it is used by multiple authors (Boon et al., 2009; Sänger et al., 2011, 2012; Chavez et al., 2003; Stam, Nolte, & Daffertshofer, 2007; Lindenberger et al., 2009; Szymanski et al., 2017) and I agree with Boon et al. (2009) that it reflects most precisely the nature of the measure. To compute the PLI, we start with a complex-valued signal c (like the Fourier transform of a signal) and obtain the phase φ analytically:

$$\varphi \equiv Im(\ln(c)) \tag{3}$$

One advantage of using a complex-valued signal is that we can represent a wave as a vector on the unit circle. This allows for simple calculations with regards to the phase differences. For two signals c_1 and c_2 with fundamental frequencies f_1 and f_2 such that $f_1 \approx f_2$ holds, phase synchronisation is defined as $|\varphi_1 - \varphi_2| < C$, where C is some constant (Boon et al., 2009; M. Rosenblum, Pikovsky, Kurths, Schäfer, & Tass, 2001). The intuitive idea behind this computation is very straighforward: we take comparable frequencies and simply investigate if their phases are close to each other. The amplitude of the signals is not taken into account and does not influence the quantification of the synchronisation, which is an intuitive way to look at the phenomena of synchronised oscillations. If we want to know if two oscillators are coupled, we normally don't care about their amplitude to determine their level of synchronisation. The cyclic relative phase is defined as:

$$\Psi \equiv (n\varphi_1 - m\varphi_2) mod 2\pi \tag{4}$$

If the two signals are unsynchronised, the phase differences over a longer period of time will follow a uniform distribution on the unit circle. Any peak in the distribution of Ψ can be understood as an indication of phase synchronisation (M. Rosenblum et al., 2001; Boon et al., 2009). This distribution can be quantified as the Phase Locking Index (PLI), which is defined as:

$$\gamma \equiv \left| \left\langle e^{i\psi[k]} \right\rangle_k \right| \tag{5}$$

where $k = \{1, \ldots, K\}$ is a discrete time index, K is the total number of samples, $\langle . \rangle_k$ means the time average and i is the complex number $\sqrt{-1}$. The variables n and m can be any integer for the general purpose of calculating the phase locking between coupled oscillators (M. Rosenblum et al., 2000, 2001) but because we will only compare similar frequencies between both Fourier series in our context we take $n = m = 1$. When there is a strong synchronisation between the signals, γ will be close to one while it will be close to zero if there is no synchronisation. This effect is obtained by adding the complex vectors and taking their mean value. A property of complex vectors is that they will cancel each other out to the level that their phases are opposite. A uniform distribution of phase angles will thus lead to a vector length close to zero.

An alternative approach

The possiblity of false positives

While the concept of phase locking as defined in equation (5) might be sound in the general context of coupled oscillators and while the PLI is positively correlated with different types of teamwork as defined under various research designs, this section of the paper will argue why there is a good reason to modify the equation when we want to measure neural synchronisation. The current definition of equation (5) could lead to false positives on neural synchronisation, because it can not discriminate between the presence or absence of neural synchronisation under certain assumptions. These two assumptions are: (I) specific states of mind imply a specific frequency band and (II) Phase Stability.

States of mind imply a frequency band. One of the previously mentioned conclusions from the research with the power spectral analysis is that specific states of mind and activities imply the brain to oscillate relatively more within specific frequency bands. For example, let's consider the research done on drawing and clay scuplting by Kruk et al. (2014) and on dancing by Fink et al. (2009). While the participants were drawing, claying or dancing the research found that they produced more waves in a specific frequency band, namely the gamma frequency band which lies between 25 and 30 Hz. Participants will thus have a larger amount of frequencies within the same bandwidth than before they started the activity. This means that the similarity between the distribution of frequencies in their power spectral analysis will increase, simply by performing a similar activity. These two papers are particulary interesting, because their research design is somewhat comparable to the designs that intend to measure neural synchronisation. We could wonder if those participants would have also shown neural synchronisation while dancing or claying together, in addition to having a similar power spectrum.

Phase Stability. For the second assumption, we need to define the property of Phase Stability:

$$\Phi \equiv \varphi(t_0) - \varphi(t_k) \tag{6}$$

where $\varphi(t)$ is the phase of the signal at time t, t_0 is the start time of the signal, $k = \{1/f, \ldots, K/f\}$ is a discrete time index in seconds with f the frequency of the signal, $\{1, \ldots, K\} \in \mathbb{N}$ and K/f the end time of the signal. If it holds that the distribution of Φ follows a normal distribution with a mean $\mu \approx 0$ and a standard deviation $\sigma < \epsilon$ with ϵ a small threshold value with regards to the unit circle, the signal has Phase Stability. The definition of Phase Stability does not require any interaction between two signals. This means that a signal can obtain Phase Stability independent of another signal, something that could possibly be caused by the introduction of an activity that invokes a specific state of mind.

As we now have defined the necessary assumptions, let us consider two cases A and B. Lets assume that participants in both cases join an experiment where during the control condition they have an arbitrary state of mind without a lot of Phase Stability and during the test condition they shift to a specific state of mind with increased Phase Stability. Under assumption (I) we can conclude that shifting to a specific state of mind implies the brains of all participants shift to a more similar distribution of frequencies as compared to the control condition. This means that there will be more overlapping frequencies that compared with the experimental condition. Now let's assume that the difference between the cases A and B is that in case A the two participants have absolutely no interaction with each other and it is impossible for them to reach neural synchronisation. Lets assume that in case B the participants will reach complete neural synchronisation. For the measurement of neural synchronisation to be sound we would need to see that it is impossible to measure neural synchronisation under condition A and that we can discriminate it from case B that meets all the conditions for neural synchronisation. If we would find neural synchronisation under condition A we will consider this to be a false positive.

In case A the complex-valued signals c_1 and c_2 of two arbitraty participants will have a more similar distribution of frequencies during testconditions because of assumption (I). This leads to a larger chance of measuring overlapping frequencies in the Fourier series of the signal that can be compared between the two signals. At the same time they will have an arbitrary nonzero difference between their phases, because we assumed there is no neural synchronisation. The resulting value of Ψ will thus be a arbitraty nonzero value and result in a vector $e^{i\Psi}$ that has a phase equal to the phase difference between φ_1 and φ_2. Because we assumed Phase Stability, computing the mean vector will result in a vector with a length close to one and an *arbitraty nonzero* phase angle. Because equation (5) only considers the length of the vector, the value of γ will be close to one and thus indicate neural synchronisation in condition A. Because we only assumed (I) and (II), but no neural synchronisation, we will consider this a false positive. Ignoring the phase value of the resulting mean vector is a sound thing to do in the context of pure physical systems of coupled oscillators like pendulums or planets (M. G. Rosenblum, Pikovsky, & Kurths, 1996). The only thing we want to do in that context is to establish *any* significant statistical coupling between two oscillators.

However, it has been shown that this approach does not give sound results in the context of EEG signals because we can have an alternative cause of the phase locking we would want to exclude.

Neural Synchronisation Vector

Now let's look at case B and see if it is possible to discriminate the false positive in case A from the true positive in case B. Let's assume the two complex-valued signals c_1 and c_2 of two arbitrary participants have a phase difference close to zero because they reached complete neural synchronisation. The resulting value of Ψ will then be close to zero and under the assumption of Phase Stability the value of the mean vector will have a length close to one and a *zero* phase angle. They key to discriminate between the cases A and B lies in the phase angles: in case A the phase angle of the mean vector will have an arbitrary value, while in case B this value will be close to zero. This means we will need to modify equation (5) such that we keep the phase information of the mean vector intact and obtain a *Neural Synchronisation Vector* (NSV):

$$\vec{\gamma} \equiv \left\langle e^{i\psi[k]} \right\rangle_k \tag{7}$$

When there is a strong synchronisation between the signals the length of $\vec{\gamma}$ will be close to one and the distribution of the phase angle will not follow a uniform distribution but will have a fixed value. If there is no synchronisation the phase angle will have an arbitrary value. Depending on whether we have Phase Stability, we will obtain a length of $\vec{\gamma}$ close to one regardless if there is neural synchronisation. This is possible because Phase Stability is a property of one signal that does not require any interaction with another signal. This lack of interaction implies that on the basis of the assumption of Phase Stability, we can only expect the phase angles of $g\vec{am}ma$ to follow a uniform distribution. If the phase angle of $g\vec{am}ma$ does not follow a uniform distribution, but converges towards zero instead, this means there has to be a certain form of interaction between the phases in order to diverge from the uniform distribution of the phase angle.

Conclusion

Coupled oscillators in general and more specific neural synchronisation are fascinating subjects. To properly test the hypothesis that neural synchronisation is a neural substrate for teamwork it might sound obvious that we have a proper control condition and that our measurements exclude false positives. However, in practice these things are not as straightforward as it might seem. There are many subtleties involved with both the research design and the mathematics of the measurement. And while the point of the control condition has been made before, the critique on the mathematical technique seems to be new. The second section of this paper showed that there are assumptions under which the PLI will lead to a false positive, something that can be avoided by using the NSV. It could be interesting to re-examine the existing datasets with the NSV in order to find out whether this will lead to a different outcome. Before introducing neurofeedback, it would be desirable to understand what it exactly is that the neurofeedback is trying to reinforce.

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