Apriority is dead: why all statements should be open to revision

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A priori truths are defined as statements to be confirmed no matter what (Putnam, 2013, p.481). They are opposed to a posteriori truths that need an experimental check to determine their truth-value. In the paper 'Two Dogmas of Empiricism' Quine (1951, 2013) rejects the distinction between a priori and a posteriori truths by claiming that *no statement is immune to revision*. In this paper I will start by exploring the importance of this statement. Secondly I will look into the critique on this statement of Grice and Strawson (1956, 2013) and the defense of Putnam (1976, 2013). Finally, after introducing Kripke models as a formal notation, I will add my own arguments for the rejection of apriority. I will do so by expanding on Quine's sidemark about the philosophical implications of developments in modern physics and conclude that even classical examples of a priori truths turn out to be a posteriori statements on closer inspection.

Importance of the statement

A priori reasoning blocks innovation

As Putnam (2013, p.479) notes, Quine (2013) attacks the distinction between a priori and a posteriori truths in different ways, for slightly different notions of the distinction. In this paper I will focus on just one of these notions, classified by Putnam as dealing with "immunity to revision". Before looking into the details, I want to point out the overall importance of the statement Quine makes. According to Putnam, Quine is of historic importance "because he was the first philosopher of the top rank both to reject the notion of apriority and at least to sketch an intelligible conception of methodology without apriority" (Putnam, 2013, p.479). While this can be considered to be an underestimation of non-Western philosophy[1], it is probably true that Quine is the first philosopher with this view that was taken seriously in Western philosophy. But why would a philosophical discussion about the existence of a priori statements be relevant to anyone besides philosophical scholars? Putnam points out that if there are truths that are *confirmed not matter what*, then "these are simply truth which it is *always rational to believe*, nay, more, truths which it is never rational to even begin to doubt" (Putnam, 2013, p.481).

While this might seem to be a detail to some, I want to argue that the intellectual freedom to doubt and experimentally test every statement is a cornerstone specifically of scientific research and generally of innovation and creativity. Marking certain ideas to be "beyond doubt" excludes complete intellectual landscapes from research. When we are right about labeling something an a priori truth, it could potentially save us all the time we might have spend researching the opposite. Yet, if we make a mistake we introduce a fundamental blockage to innovation, creativity and scientific progress. This implies we ought to be very carefull before we incorporate this division into our reasoning. If we look at previous scientific revolutions, most involved revising principles that were once regarded to be a priori statements. The irony is that the ideas we now consier to be genius breakthroughs often were initially ridiculed, prosecuted and even forbidden by their contemporaries. And even though we nowadays look back at the prosecution of Galileo by the church as a dogmatic mistake [2] we simultaneously reintroduce (probably more accuracte: failed to remove) the dogmatic foundation that allows this type of reasoning by the belief in a priori statements.

Mistaking common sense for a priori truths

How can people decline and even ridicule a hypothesis, before there is any experimental evidence that contradicts it? Where, after the experimental evidence finally comes in, the hypothesis turns out to be confirmed? I want to argue here that statements seemingly being confirmed by common sense is an important motivation for people to erroneously believe them to be a priori truths. If there is any substantiation at all, these statements are often 'substantiated' by phrases like "it needs no further explanation that..." or "it is clear that..." They are rarely backed up with formal proofs, because they do not seem to need them. Intuitions based on common sense are generalised experiences, hypotheses that are constructed on the basis of the ongoing 'experiment' of daily life. It is an everyday experience that everything stops moving if you throw it away. This is why we needed Newton to formulate

[1] It can be defended that Buddhist philosophy rejected a priori statements millennia before Quine did, with the statement that it is possible that an argument is both well-reasoned and false, and thus that "the soundness of reasoning is no guarantee of truth" (Jayatilleke, 2013, p. 273).

[2] for which the church eventually apologized (Pope John Paul II, 1992)

the first law of motion in 1687: an object remains at rest or continues to move at a constant velocity, unless acted upon by a force (Newton, 1999). Why didn't people come up with this idea before? And even considered the opposite to be an a priori truth? If we throw something, it stops moving after a while. Thus their daily experience, and thus their common sense and intuition, contradicted Newton's law. History shows that we have a habit of mistaking the generalised results of our common sense experiments for a priori truths. This is not a type of mistake we have left behind since the scientific revolution. Just consider the fact that scientists that researched the foundations of quantum mechanics in the last century did not mention this to colleagues because they were afraid that it would have a negative impact on their career (Carroll & Becker, 2019; Carroll & Reid, 2019). The editor of the Physical Review, a major journal, even sent out a memo at some point to forbid publications on the foundations of quantum mechanics (Carroll & Reid, 2019). During the second half of the 20th century, quantum mechanics was just thought to be "too weird" to seriously consider.

An increasing gap

The scientific development of the past century has increasingly broadened the gap between common sense and the scientific worldview, especially in the domain of physics. This implies that holding on to our common sense (and confusing it with a priori truth) will increasingly become an obstacle for the progress of knowledge. Our five senses are simply not the best tools to explore the deeper truths of reality. Consider Sean Carroll and Leonard Susskind discussing their worldview (Carroll & Sussskind, 2019). They are both professors in theoretical physicis. Susskind thinks that the *most plausible* hypothesis, given the current experimental evidence, is that our universe consists of vibrating 10-dimensional strings. Carroll thinks that the *most plausible* explanation of the experimental results of quantum mechanics is that our universe constantly branches of into parallel universes. Note the word plausible here. Obviously, their intuition for what is plausible has deviated a lot from what is common sense, something they point out themselves. The experiences of parallel universes or 10-dimensional strings are not directly accesible from our normal states of consciousness. Until quite recent people proposing there are parallel universes with slightly different copies of you reading these same words simultaneously would be considered confused at best or mentally ill and in need of medication at worst in our Western culture. This illustrates how scientific insight increasingly conflicts with our intuition and common sense. Listening to the dialogue of Carroll and Susskind will violate the conceptions of what most people assume to be a priori truths. Because of the complexity of the mathematics behind the theories, most people are not in a position to verify for themselves what they think to be the most 'plausible' given the experimental results. This is probably a reason why these scientific insights are taking so long to enter the public discourse, considering the first ideas about quantum mechanics go back almost a century. But rejecting these theories exclusively on the basis of common sense disguised as a priori reasoning shouldn't be considered a scientifically sound option.

Critique: defending a dogma

Grice and Strawson (1956, 2013) critique the paper of Quine and defend the division between analytic and a priori statements. Their paper runs along the various arguments Quine gives against different notions of analytic and a priori statements.

Tradition of use

Their first counterargument is the tradition of use. According to their reasoning, the mere existence of a tradition of use "seems to suggest that it is absurd, even senseless, to say that there is no such distinction" (Grice & Strawson, 2013, p.470), especially because the cases do not form a closed list. The authors display a huge confidence in the collective wisdom of philosophers and seem to forget that *most* scientific revolutions are an example of people finding out that their tradition was mistaken. This implies that there isn't necessarily an overlap between the collective perception of humans and the truth. In fact the biggest scientific revolutions show how easily our collective perception of the world is misguided, as I pointed out in the first section.

Two classes of matter. To show how their argument of tradition fails, I will give a counterexample. In the time of Galileo people made a distinction between two classes of matter. They classified 'heavenly matter' like the planets and 'earthly matter' like rocks, while these objects certainly did not form a closed list. These classes obeyed different sets of laws and this distinction was the traditional explanation why heavenly objects like the moon did not fell down while earthly objects did (Galilei, 1710, 2012). But since Newton we accept that this distinction does not exist, and that all matter is governed by the same laws of gravity. So even though this classification of an open list had a tradition of use, it turned out that no such distinction exists. I conclude that a tradition of use does not imply the truth of a distinction. It seems that Grice and Strawson (1956) fell into the trap of mistaking commons sense for a priori truths. By giving the tradition argument the authors illustrate one of the effects of this mistake: they are defending the status quo instead of promotion a curious and open mind towards unthought explanations.

Revision as a shift in sense

As I referred to earlier, Putnam (1976, 2013) points out that Quine attacks different notions of truth with different types of arguments. He labels one type the "linguistic" notion of analitycity and another the "notion of an analytic truth as one that is *confirmed no matter what*" (Putnam, 2013, p.479). The latter is one of the traditional notions of apriority. Putnam goes on to explain that the he considers the latter to be the most important and that, while Quine says that there is no distinction between analytic and synthetic truths, he ought to have said that there is no distinction between a priori and a posteriori truths. Grice and Strawson (1956, 2013) adopt the merging of the concepts 'analytic' and 'a priori' by saying that "Quine's objection is not simply to the words 'analytic' and 'synthetic', but to a distinction which they are supposed to express, [. . .] by means of such pairs of words or phrases as 'necessary' and 'contingent', 'a priori' and 'emperical', 'truth of reason' and 'truth of fact' (Grice & Strawson, 2013, p.470)". Their paper gives a lot of counterarguments against the "linguistic" arguments of Quine. I will ignore those because I restricted my paper to the notion that deals with "immunity to revision" and will thus focus on their explicit arguments against this idea. They argue that the only way to make sense of the idea of conceptual revision is by shifting the sense in which the words are used: "If we can make sense of the idea that the same form of words, taken in one way (or bearing one sense), may express something true, and taken in another way (or bearing another sense), may express something false, then we can make sense of the idea of conceptual revision." (Grice & Strawson, 2013, p.477). In addition to this they conclude an example of a logical impossibility by saying that unless someone agrees on a shift in the sense of the meaning of the words "we shall say, not that we don't believe him, but that his words have no sense" (Grice & Strawson, 2013, p.474). My understanding of their position is that all revisions can be sufficiently explained as different meanings being attached to the same words. This implies that what was true in one sense, will still be true after the revision. We didn't actually revise a priori concepts but we just mean something else. It is only on the superficial level that the words are the same. In the rest of this paper I will give counterexamples against this "all revision are a shift in sense" argument.

Notions of truth

While both Quine, Grice and Strawson mix the notions of analyticity and apriority, this can be a cause of confusion. For example, within the boundaries of a certain framework or model, things can simply be defined to be true. This is not a truth "no matter what", but a truth within a model and thus under the assumptions of the model. It is possible to accept this as the definition of a necessary truth, while rejecting the notion of truth "no matter what". To avoid this confusion I felt the need to start with giving clear definitions that discern between the two notions of truth. For reasons of brevity, I will not dive into the complete history of the concepts of analyticity and apriority, but will simply define a framework that helps to clarify my argument.

Truth, given a model of reality

Kripke models. I agree with the idea that there are statements that are always true under the assumption of a specific model. It is fairly easy to construct a model with certain rules, in which things are necessary true given that those rules are followed. I will use the formalism of Kripke (Van Ditmarsch, van Der Hoek, & Kooi, 2007) to describe the idea that a statement can be true given one model, but false given another model. Kripke formalised this notion by introducting the concpet of a Kripke frame $\mathcal{F}$, which is defined as:

$$\mathcal{F} = \langle \mathcal{W}, \mathcal{R} \rangle$$

where $\mathcal{W}$ is a set of possible worlds or states and $\mathcal{R} \subseteq \mathcal{W} \times \mathcal{W}$ is a set binary relations on $\mathcal{W}$, also know as the accessibility relation. These relations describe how we can make transitions from one world or state to another. This transition can be understood as movement along temporal, spatial or even imaginary dimensions. To give an example in the temporal domain: the night can be considered one possible state of the world, the daytime another. Our world makes transitions between these two states about every 12 hours. A Kripke model $\mathcal{M}$ is defined through the extension of a Kripke frame by adding a function $\mathcal{V}(w)$ that gives truthvalues for statements in the different worlds:

$$\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{V} \rangle$$

Where $\mathcal{V}\colon \mathcal{W} \rightarrow \mathcal{P}(\Phi)$ is a function that maps worlds $w \in \mathcal{W}$ and true statements $\varphi \in \Phi$, where $\mathcal{P}(\Phi)$ is the powerset of all possible true statements. The function $\mathcal{V}(w)$ tells us which subset of propositions is true in world w. In the example of worlds with day and night, this could mean that while it is true that stores are open during the daytime, this can be false during the night. We can now denote for a certain statement φ that it is always true in a certain model $\mathcal{M}$:

$$\mathcal{M} \models \varphi$$

This implies that in the model $\mathcal{M}$ for all worlds $w \in \mathcal{W}$ the proposition φ is true, eg:

$$\mathcal{M}, w \models \varphi$$

Playing by the rules. To make this a bit less formal, let's take an arbitrary board game, e.g. Monopoly, and incorporate the official Monopoly rules into our model of the world $\mathcal{M}$. Now if a player wants to play the game, one of the rules is that he has to choose one of the available ten tokens. Given this model, and given that a player follows the rules of the model, we can know for sure that a player will always have one of these tokens, and not two, or some other tokens. Given this specific model, these can be said to be necessary truths.

It is immediately clear that while this can be defined as a necessary truth, it is not an apriori truth "no matter what". It is easy to imagine a counterexample where an eleventh token is added to the game[3]. I deliberately choose the example to be as mundane as possible. There is no need for a Platonic aura surrounding necessary truths and in this example the truth of the statement is clearly the consequence of a mental construction, not of the fabric of reality itself. Besides that, the model and statements are clearly open to revision. We could update the model as new experimental observations come in (e.g. finding a version of Monopoly with an extra token), or when we have better explanations for the same phenomena (e.g. the official rules were updated). It could also be the case that we find out that our model has some implicit characteristics that we need to specify in order to close some loopholes.

With this notation we can describe how statements can be necessary truths under certain conditions. This allows us to make formal proofs within a given framework of assumptions. This implies that we will have to say something about the model under which statements are necessary truths. Experimental evidence could always force us to modify that model of the world by specifying formerly implicit rules, adding new ones or modifying existing rules. This is exactly what is thought to be impossible for a priori truths, because those ought to be true "no matter what". Most importantly, it emphasizes that necessity under a model is an artificial artefact that only exists within formally defined conceptual boundaries of the model.

Omitting the model. With this formal notation, we can describe what conditions a statement would need to fulfill if it is an a priori truth. An a priori truth is suggested to be true regardless of the model, or the model is beyond any doubt which implies there is only one possible model which is similar to reality itself and that no experimental evidence can change. We can denote this, using the Kripke notation, by emitting the model $\mathcal{M}$:

$$\models \varphi$$

This would mean that the statement φ is always true, for all frames $\mathcal{F}$ and all models $\mathcal{M}$ (Van Ditmarsch et al., 2007). If we can show that φ is true for some worlds, but not for others, we show that we can not omit the model $\mathcal{M}$ but need to specify the conditions under which the statement is true or false. This would proof that the statement is not an a priori truth "no matter what", but true only under conditions specified in the model $\mathcal{M}$.

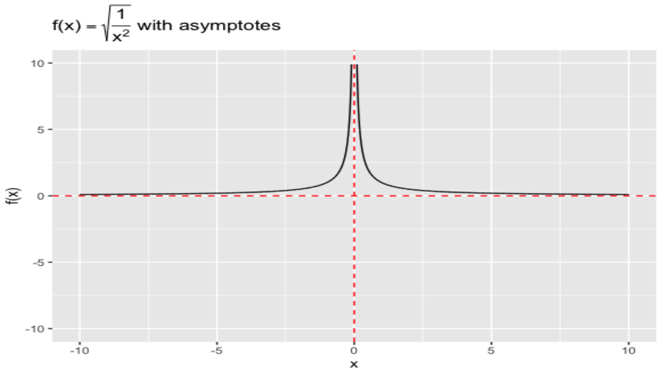

Figure 1. Function with asymptotes in red

Truth in the world

We can contrast the notion of "truth, given a model" with an idealised "truth in the world". The scientific models we try to create are approximations of this idealised truth that is thought to be "out there". We know that Newtons laws of gravity are a fairly good approximation of reality (assuming that something as reality exists), but we also accept that the models of Einstein are a better approximation of reality. Where "truth, given a model of reality" can be compared to a function, "truth in the world" can be considered to be the asymptote of the function. A function can come arbitrarily close to its asymptote, but will never completely overlap (see figure 1. The idea that we are able to identify a priori truths implies that we are able to know how to describe the asymptote. The idea that it is impossible to identify a priori statements implies that we are fundamentally unable to approximate how close we have approached the asymptote. Even though our models of the world might come infinitesimal close to the truth in the world with an negligible error, we can never be sure. To quote Putnam (Putnam, 2013): "We never have an absolute guarantee that we are right, even when we are."

Two counterexamples

A priori, or maybe not?

With the formal definitions in place, let's examine two typical examples that are given when analytic or a priori statements are illustrated. These typical examples are not clearly false under a different model, as was the case with the mundane example I gave regarding Monopoly tokens. At first sight, these examples appear to be independent of a specific model of the world. They seem to rely on the structure of the world itself and are thus suggested to be beyond the

[3]During the history of Monopoly, the amount and type of tokens actually changed frequently: `https://monopoly.fandom.com/wiki/Tokens`

need for revision unless the fabric of reality itself changes. This is a matter of course because the whole point of the these typical examples is to show how some statements are indepent of our model of reality. The two examples are:

(1) All squares have four sides.

(2) No one is simultaneously bachelor and married.

For a square, we use the Euclidian definition of an equilateral and right-angled figure. If we denote the first statement as φ_4 (with the subscript denoting the amount of sides on the square) and the second statement as ψ, we can write these statements formally as:

$$\models \varphi_4 \quad (1)$$

$$\models \psi \quad (2)$$

Because there is no model $\mathcal{M}$ defined, this notation suggests that the statements φ_4 and ψ do not require certain conditions specified by a model before the statements are true. It suggests that we don't need to check some observations before we can confirm that a square has four sides, or that a bachelor isn't married at the same time. At first sight, these claims seem to be acceptable as a priori truths. Most people are convinced they have never seen a counterexample and thus their common sense tells them that these things should always be true. They probably will have a hard time imagining a possible counterexample how these things could *not* be the case. Their reaction might even resemble the statement Grice and Strawson (2013, p.474) make while discussing a logical impossibility: "For unless [the person making seemingly logical impossible claims] is prepared to admit that he is using words in a figurative or unusual sense, we shall say, not that we don't believe him, but that his words have *no* sense." This aligns with their argument against the possible revision of a priori arguments. Grice and Strawson (2013) suggest that the only imaginable way that a seemingly logical impossible statement can be true, is by shifting the meaning of the words in such a way that we mean something different and thereby avoid the contradition. They simply refuse to consider that they might need to revise their model of reality in order to make sense of the statement.

However, if I can show that we actually need to define a conditional model $\mathcal{M}$ because there are situations for which these statements are false, I will consider that to be a counterexample for the statement that these examples are true "no matter what". Stated slightly different, showing that these statement are only true under certain local conditions (without shifting the sense of the words) is a valid counterexample against the suggested universal necessity. In the next sections I will show the reader how these two examples are indeed dependend on certain models of geometry and logic. In addition to that, I will show that these models have actually been shown to be false or at least in need of revision through certain experimental results in physics. The inability to make sense of a contradictionary statment can also be caused by having a model of reality that is too restrictive, instead of exclusively being explained as either a shift in meaning or the person giving the statement talking gibberish. My conclusion will be that we have a habit of overestimating our common sense and as a result confuse it with a priori truths.

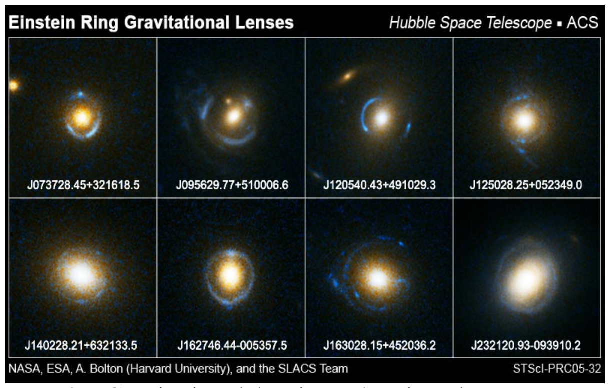

Figure 2. Gravitational lensing, showing the curvature of spacetime

On Earth, all squares have five sides

The statement that all square have four sides, assumes Euclidian geometry. Euclidian geometry is described in Euclids book "The Elements" (Fitzpatrick, 2007; Playfair, 1814), written around 300 BC. It contains five axioms on which Euclid builds the rest of his geometry. Euclid gives us a definition of a square in book I, definition 22: "a square is that which is both equilateral and right-angled". If our model of reality includes the five axioms of Euclid, it will necessarily be the case that any equilateral and right-angled figure has four sides.

Spacetime curvature. To understand why not all five of the Euclidian axioms are valid in our universe, we need to consider the nature of space. The General Theory of Relativity states that gravity should be considered to be the curvature of spacetime[4] (Einstein, 2018; Einstein et al., 1916; Torretti, 2000; Bahamonde & Faizal, 2019). This curvature of spacetime is experimentally illustrated by observations of Einstein-Chwolson rings (Perlick, 2004; Lee, 2017), which is the bending of light from distant stars through gravitational lensing caused by the curvature of spacetime around these stars (see figure 2). Due to the gravitational field of our Sun and Earth the spacetime of Earth is slightly curved. This means that the curvature of our spacetime is not com-

[4]Hermann Minkowski introduced the idea of combining space and time into a four-dimensional "Minkowski space" which is normally referred to as spacetime. This laid the mathematical foundation for Einsteins special theory of relativity (The Editors of Encyclopaedia Britannica, n.d.)

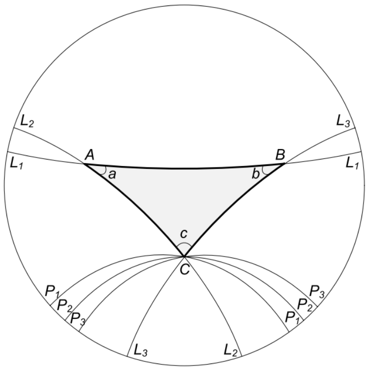

Figure 3. Lines $P_{1,2,3}$ are all parallel to L_1 (Krioukov et al., 2010).

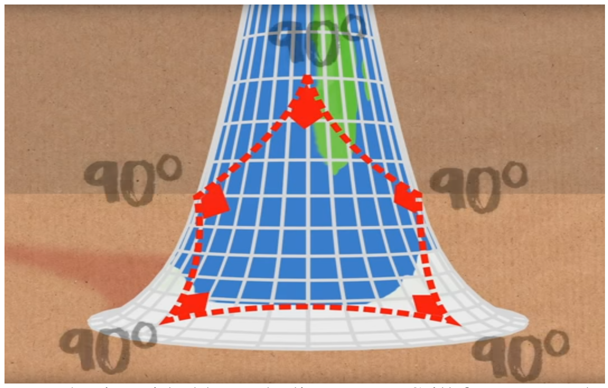

Figure 4. Five sided hyperbolic square. Still from a numberphile video (Stoll, 2018)

pletely flat and thus does not have an Euclidian geometry, but a hyperbolic geometry.

Hyperbolic geometry. This type of geometry was developed independently by both Lobachevsky and Bolyai [5] in the 19th century (Bolyai, 1896; Lobachevskiĭ, 1891). One of the essential characteristics of hyperbolic geometry is that the fifth axiom of Euclid no longer holds. This is the axiom that deals with parallel lines. A short (but logically equivalent) version of this postulate is formulated by Playfair (1814): "In a plane, given a line and a point not on it, at most one line parallel to the given line can be drawn through the point.” In a hyperbolic geometry it is the case that an infinite number of parallel lines can be drawn through a point not on a line (Krioukov, Papadopoulos, Kitsak, Vahdat, & Boguná, 2010; Stoll, 2018). This is illustrated in figure 3. A fundamental problem in visualising hyperbolic geometry is that our minds are trained to work with Euclidian geometry. So we will need to use Euclidian geometry to visualise hyperbolic geometry, which has fundamental difficulties because hyperbolic space is “larger” than Euclidian space and thus wil never fit. Imagine trying to fit a square into a circle, or to draw a 3 dimensional figure on a 2 dimensional plane. The only way to do this is to distort the representation, so keep in mind that every representation can emphasize another part of the hyperbolic geometry but can never visualise all properties simultaneously (Krioukov et al., 2010).

Imagining hyperbolic space. To help the reader imagine a hyperbolic curvature, lets start with the easier case where spacetime is curved spherical like a globe. We can make a 90 degree angle on top of the sphere (the “North Pole”) and two 90 degree angles on the “Equator” of the sphere. Because spacetime itself is curved, these lines seems to be curved lines from our perspective but will actually be straight lines and thus the shortest paths between the two points from the perspective of an observer that lives inside the curved spacetime. The lines would not look curved, because our vision would follow the curvature of spacetime. Standing on the equator, we would be able to look along a straight line and see the North Pole at the end of this straight line. This is not a shift in the meaning of a straight line. If we would deny that these are straight lines, we are actually shifting the sense of what a straight line is (namely, the shortest path between two points). We are thus able to construct a figure that is both equilateral and right-angled and thus follows the Euclidian definition of a square, while it has only three sides. On a hyperbolic curvature, this effect is exactly the other way around. A hyperbolic curvature can be imagined to look like a funnel. In a hyperbolic geometry, a figure that is both equilateral and right-angled will have five sides, and by the Euclidian definition we should call this a square (see figure 4). We normally do not notice that our spacetime has a hyperbolic curvature, because the curvature is very small. However, it is still a fact that our space has a hyperbolic curvature due to the gravity of our Sun.

The implications. This implies that on Earth four-sided and equilateral figures can not exactly be right-angled, or that all equilateral and right-angled figures will have five sides. Admittedly, the deviation from the 90 degree angles will be extremely small, but it will be measurable. This means we are forced to replace statement (1) and specify our model of reality:

$$\mathcal{M}_E \models \varphi_4 \tag{3}$$

$$\mathcal{M}_H \models \varphi_5 \tag{4}$$

Where $\mathcal{M}_E$ is the Euclidian model of our spacetime, and $\mathcal{M}_H$ is the hyperbolic model. The subscript i on the proposition φ_i stands for the amount of sides a square has. In plain English we could say that if we follow an Euclidian model $\mathcal{M}_E$, all equilateral and right-angled figures have four sides. If we follow a hyperbolic model of geometry $\mathcal{M}_H$, all equilateral and right-angled figures have five sides.

[5]Interestingly, Bolyai states in the title of his paper that Euclid’s axiom can not be decided a priori.

This means we need to revise our statement that all squares have four sides and thus the statement was not a priori. We were misleaded by our common sense when we accepted that this was an a priori truth. Even worse, the statement turned out to be actually false for our situation here on Earth. The only way to get four-sided squares is by either shifting the meaning of being right-angled and accept angles slightly smaller than 90 degrees as being right-angled or by shifting the meaning of a straight line. The latter might seem a simple fix, but would complicate things even more. If we could take a shorter path while light does not, this would allow for faster-than-light travel which would give rise to time travel and would destroy causality (Giustina et al., 2015; Šafránek, 2019). Four sided squares are just convenient approximations for our Euclidian brains, idealisations of reality that conveniently fit the way we think of the world. A four sided square is good enough for most situations, but it should not make claims on being an a priori truth. And even though there might exist locations in spacetime where the geometry will follow $\mathcal{M}_E$, Earth is not among those places.

I just gave a counterexample for the argument of (Grice & Strawson, 2013) that the only way to make sense of a conceptual revision is through a shift in sense. A square is still equilateral and right-angled. Equilateral still means that every side has the same length, right-angled still means 90 degrees. Four still means four. It just is not true because we implicitly assumed an Euclidian geometry and we happen not to live in one[6].

Being simultaneously a bachelor and married

The statement "No one is simultaneously bachelor and married" or variations thereon are another classic example of an analytic statement. I will ingore the problems that Quine points out in determining the synonymy of 'bachelor' = 'unmarried man' (because those fall under the "linguistic" notion of a priority) and concentrate solely on the seemingly logical impossibility of the statement. This is what gives the statement its compelling power. A man is either married or not married. The logical problem is that the underlying logical structure '$p \wedge \neg p$' is a contradiction in classical logic, also referred to as the "law of the excluded middle". It is hard to imagine someone to be two contradictory things at the same time.

Schrödingers bachelor. In order to show how the statement can be false without changing the sense of the words, I created a modified version of Schrödingers thought experiment that is known as Schrödingers cat (Schrödinger & Trimmer, 1980; Sergeevich, 2019). Let's imagine Bob, who is both a physics student and a bachelor. Alice has proposed Bob to marry her but Bob can't decide if he should say yes or no. Because he is a physics student, he decides to use the quantum mechanical equivalent of tossing a coin to decide whether he should marry Alice. So Bob sets up a big black box, in which he puts the double slit experiment, a classical quantum mechanical experiment. In this experiment just one photon will be omitted. This photon will be beamed towards a double slit. The black box is big enough for both Bob and Alice to sit inside. If the photon goes through the upper slit, Bob will say no. In the case that the photon goes through the lower slit, Bob will say yes and he will be a married man.

Entangling Bob. If we use a quantum mechanical notation (Sergeevich, 2019), we can denote the photon going through the upper slit as $|1\rangle$ and the photon going through the lower slit as $|0\rangle$. We describe the state of the photon as a wave function, where both states are possible: $(\,|1\rangle + |0\rangle\,)$. Now let's fire the photon. For us, standing outside the black box, we don't know wether the photon went trough the upper slit or through the lower slit. We say that the wave function of the photon didn't collapse, and we still denote the state of the photon as $(\,|1\rangle + |0\rangle\,)$. For Bob, something different happens. He has also a wave function, which we will denote as $|\beta\rangle$ when he is a bachelor, and as $|\mu\rangle$ when he is married. From his perspective the photon either chooses the lower path or the upper path. This means the wave function of Bob and the photon will interact, something we call 'entanglement'. We describe this transformation like this:

$$|\beta\rangle\,(\,|1\rangle + |0\rangle\,) \rightarrow |\beta\rangle\,|1\rangle + |\mu\rangle\,|0\rangle$$

There are two possible state after the firing of the photon. One is the state where the photon went through the upper slit $(\,|1\rangle\,)$ and Bob said no, so he is still a bachelor $(\,|\beta\rangle\,)$ and we can describe the state of that system as $|\beta\rangle\,|1\rangle$. The other situation is that the photon went through the lower slit $(\,|0\rangle\,)$ and Bob now is a married man $(\,|\mu\rangle\,)$ which we describe as $|\mu\rangle\,|0\rangle$. But for us, the situation is different. As long as we do not observe the system, the wave function doesn't collapse and we still have to describe the system as $(\,|\beta\rangle\,|1\rangle + |\mu\rangle\,|0\rangle\,)$. We have to say that the photon went *both* through the lower and the upper slit and consequently that Bob is both a bachelor and married. When someone would ask us 'is Bob a bachelor?' we need to say 'yes'. And if someone asks 'is Bob married?', we have to say yes again. Bob is in a superposition of simultaneously being a bachelor and being married. Experimentally, large scale objects have been entangled and put into a superposition, so Bob's size should not have to be a problem (Belli et al., 2016; Kovachy et al., 2015; Arndt & Hornberger, 2014).

Looking for an emergency exit. Most people feel confused after they hear a version of Schrödingers cat for the first

[6]This also means that the Pythagorean theorem no longer holds, at least not on Earth. It is just a convenient approximation that ignores the curvature of our spacetime. So, when Frege (2013) claims in 1918 that "the thought we express by the Pythagorean theorem is surely timeless, eternal, unchangeable" he is simply wrong. A hyperbolic version of the Pythagorean theorem would be $cosh(a)cosh(b) = cosh(c)$ with $cosh(x) = \frac{e^x+e^{-x}}{2}$.

time. A lot of physicists did, too. The instinctive reaction of most people is that either they didn't understand what was said or that the physicists must have made a mistake in interpreting the results. What is actually happening is that the experiment is violating something we took to be an a priori truth. It violates our common sense and a we can understand how Grice and Strawson (1956) exclamated in the face of such logical impossibilities that this "just doesn't make sense". However, we are no longer in a position to simply dismiss it as nonsense because it makes us feel uncomfortable. We could accuse nature itself of being illogical, but a more mature reaction would be to be open up to the idea that possibly our models of reality are too restrictive and need to be revised.

Another instinctive reaction to these experiments is to try to find a way out of this madness, to find an interpretation that would keep intact what was formerly known as reality. Someone has just tried to throw a burning rag into our model of reality and naturally we hope to find an emergency exit or the fire extinguisher. Surely there must be an interpretation that can conserve our traditional model of reality we thought to be a priori true and protect us against having to revise the model? Anything that would keep us from having to follow the white rabbit into wonderland (Wikipedia contributors, 2019)? After all, if we open the black box with Bob inside, the wave function collapses and we will observe either $|\beta\rangle\,|1\rangle$ or $|\mu\rangle\,|0\rangle$. Couldn't it be the case that we are simply unaware of the *real* state of the system where the photon actually did choose a slit, and we are just unable to describe it properly?

Giving up local-realism. These emergency exits were thought (and hoped) to be the most plausible explanation for some time by the majority of the physicists. The interpretation of quantum mechanics where the photon actually does choose a slit has been labeled the local-realist position. However, in 1964 John Bell proved that "no theory of nature that obeys locality and realism can reproduce all the predictions of quantum theory" (Hensen et al., 2015). Bell described an experiment, through which physicist can discriminate between the photon having a precise (but unknown) position or not. Through this experiment, the local-realist position became an experimental question. Multiple experiments have been done, and they decisively confirmed that the local-realist interpretation can't be correct (Griffiths & Schroeter, 2018). The experiment has been repeated under different circumstances to close all possible loopholes (Giustina et al., 2015; Hensen et al., 2015). This means that there is no emergency exit and we have to give up one of both, locality or realism. Either we let go of locality and allow objects to be and act at two places simultaneously, thus allowing for "spooky action at a distance". Or we let go of realism, the assumption that reality has defined properties, independent of measurement (Zyga, 2016).

Maps of madness. Even though physicist agree that a local-realist interpretation of quantum mechanics isn't a possible interpretation of the experiments, there is no consensus on what a proper interpretation should be. As Mermin (2012) states: "Quantum mechanics is the most useful and powerful theory physicist have ever devised. Yet today, nearly 90 years after its formulation, disagreement about the meaning of the theory is stronger than ever. New interpretations appear every year. None ever disappear." The alternative interpretations all seem intuitively very weird. This is because they either have to give up locality or realism, both of them being a key ingredient in many seemingly a priori statements. Giving up either one of them will thus violate our common sense.

Cabello (2015) explores the possible interpretations in a paper titled "Interpretations of quantum theory: A map of madness". Among the serious candidates is the Many Worlds interpretation (Everett, 1973) that is gaining increasing interest (Arve, 2019). It postulates that there are new universes that split off, one in which Bob stays a bachelor and another universe in which Bob marries with Alice. Others give up classical logic and introduce quantum logic, or accept nonlocality. Eitherway, while we can hear the echo of Grice and Strawson (1956) exclamantions that "this just doesn't make sense" fade away, Google is building quantumcomputers that harvest the practical advantages of doing calculations with bits that are simultaneously a 0 and 1 (Arute et al., 2019) and performed calculations under 4 minutes that would have taken 10.000 years under the restrictions of classical logic. Lets put this in perspective. If you count one hair per second, you can roughly count all your hairs in one day if you count non-stop. Imagine you did this for for about 13.000 people during 36 years. Speeding this up with the same factor as the quantumcomputer would mean that you would need just one second for the complete process. Let that sink in for a momment: they either have speed up the computations by running them in parallel universes, or by violating the "law of the excluded middle" and thus breaking classical logic. Maybe it "just doesn't make sense" to some, but it certainly works.

The implications. Off course, one could argue that I do not completely understand quantum mechanics (which I surely do not) and that there is an interpretation of quantum mechanics that will keep all bachelors unmarried. It could even be argued that there is no problem, because in the Many Worlds interpretation Bob is simply split into two and a logical contradiction is avoided. Off course, this explanation opens the backdoor to a complete new set of illogical problems. The point is that someone will have to say: "given this specific interpretation of quantum mechanics, no one is simultaneously bachelor and married". We thus have to replace statement (2) with:

$$\mathcal{M}_i \models \psi \quad (5)$$
$$\mathcal{M}_j \models \neg\psi \quad (6)$$

Which means that under all quantum mechanical models $\mathcal{M}_\rangle$ of reality the proposition is true and under all quantum mechanical models $\mathcal{M}_|$ of reality the proposition is false. This means that statement (2) is no longer an a priori statement, but turned out to be an experimental question which shouldn't be possible for a priori statements. And proving either one of them will immediately violate many other a priori statements, because we will always have to abandon local-realism. Either way this supports my case that we can't leave the model unspecified and need to run experiments to find out what is true.

Conclusion

I defined a priori statements to follow the structure of ommiting a conditional model $\mathcal{M}$. For two statements that are classical examples of a priori statements I showed that adding this model to the statement is necessary. Even more, I showed that one of the statments is actually false, while the other statement is still an experimental question. While I have picked just two examples, quantum mechanics and relativity could easily provide us with much more disturbing counterexamples. We haven't explored distortions of time or quantum teleportation to randomly name a few candidates. Still, I hope to have convinced the reader that identifying statements as being true "no matter what" is very tricky. Instead of exclaiming that a statement "just doesn't makes sense" we could ask ourselves if our model of reality is just too restrictive. In addition to what, we should incorportate the model of reality that is conditional for the statement even though we might think this model is beyond doubt, because it probably is not. Looking at the current status of physics, we are not in a position to propose models about our reality that are "true no matter what", especially regarding the most fundamental aspects of our reality like space and time. Every model should be open to revision, which will also help to avoid the blockage of innovation because we cling to traditions and our common sense. Instead of claiming that we are able to tell which statements are absolute true, and which are not, I promote a much more humble perspective, in which we accept the boundaries of our epistemology. To quote J.B.S. Haldane: "Now, my own suspicion is that the universe is not only queerer than we suppose, but queerer than we can suppose" (Haldane, 1971). Off course, my idea is not intended to be the "one a priori statement that rules them all". I explicitly consider this to be a hypothesis, that could turn out to be proven false for certain cases. To end with the words of Putnam: "We never have an absolute guarantee that we are right, even when we are (Putnam, 2013)."

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